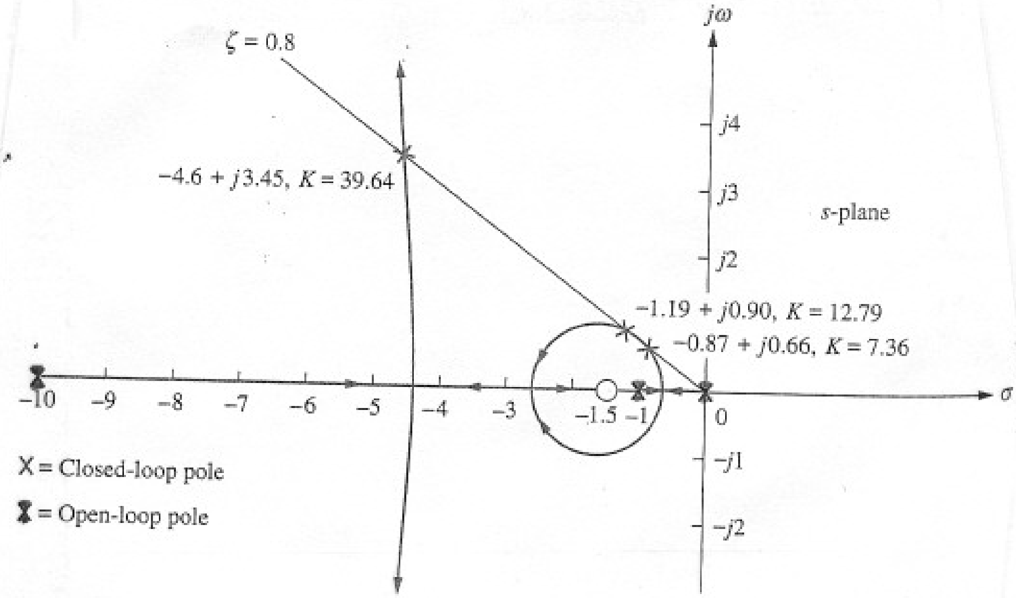
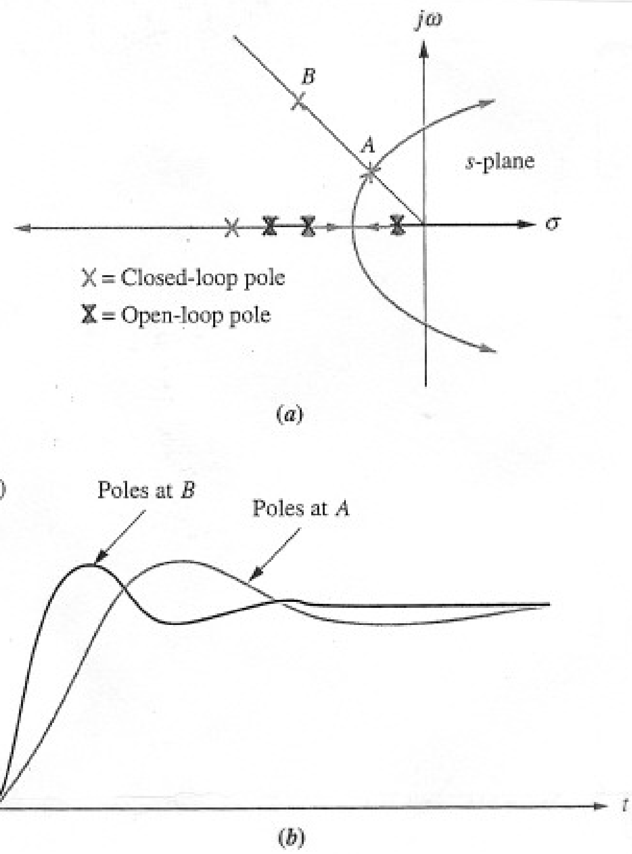


For this problem there are 3 different gains that provide an overshoot of 1.52% ($\zeta = 0.8$). But is the second order assumption valid for each of these 3 possible gains?

Case	Closed-loop poles	Closed-loop zero	Gain	Third closed-loop pole	Settling time	Peak time	K_v
1	$-0.87 \pm j0.66$	$-1.5 + j0$	7.36	-9.25	4.60	4.76	1.1
2	$-1.19 \pm j0.90$	$-1.5 + j0$	12.79	-8.61	3.36	3.49	1.9
3	$-4.60 \pm j3.45$	$-1.5 + j0$	39.64	-1.80	0.87	0.91	5.9

- For case 1 & 2 the third closed loop poles are relatively far from the closed loop zero. For these cases there is no pole-zero cancellation and a second order system approximation is not valid.
- For case 3 the third closed loop pole and zero are relatively close and the second order system approximation is valid.

So far we have seen how the root locus plots the closed loop poles as the gain is changed. However we are limited to responses that exist along the root locus



a. Sample root locus, showing possible design point via gain adjustment (*A*) and desired design point that cannot be met via simple gain adjustment (*B*); **b.** responses from poles at *A* and *B*

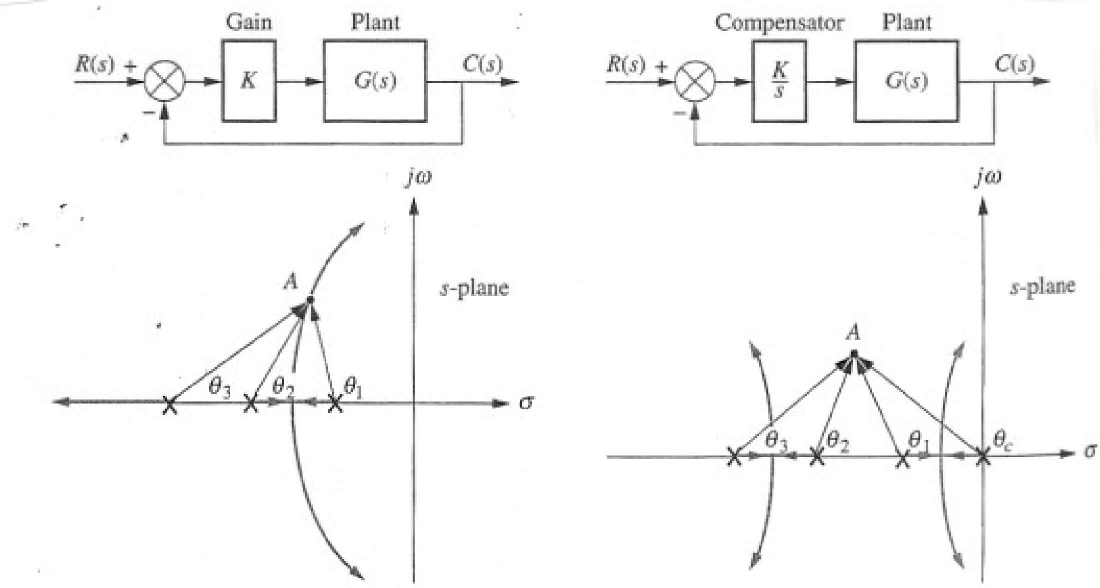
If we would like a response that corresponds with the pole location at B we can not use the present system. We have two alternatives, 1 change the plant 2 augment or compensate the system with additional poles and zeros so that the compensated system has a root locus that goes through the desired pole location for some value of gain.

Proportional Integral Compensation (PI)

We've seen that placing an open loop pole at the origin improves the steady state error. Let's consider a system with desirable dynamics except that we wish to improve the ss error.

Original System   with a pole at origin

Pole at A is:
a. on the root locus without compensator;
b. not on the root locus with compensator pole added;
(figure continues)



$-\theta_1-\theta_2-\theta_3=(2k+1)180°$

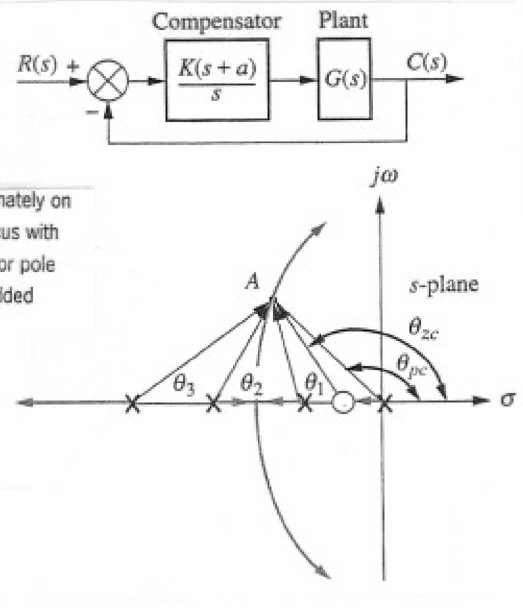
(a)

$-\theta_1-\theta_2-\theta_3-\theta_c\neq(2k+1)180°$

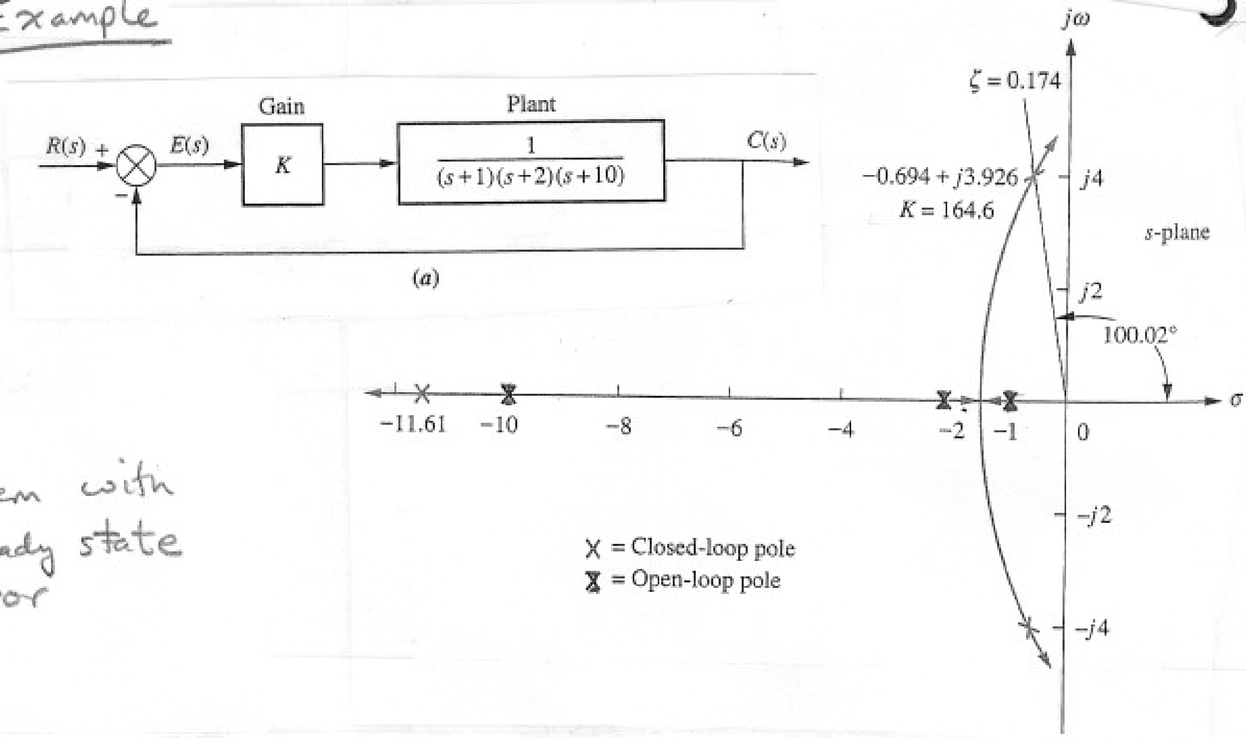
(b)

With the addition of the pole at the origin we can no longer achieve the response at pole location A. By adding a new compensator $\frac{K(s+a)}{s}$ the root locus changes and so we can obtain the desired response at A and have improved steady state error!

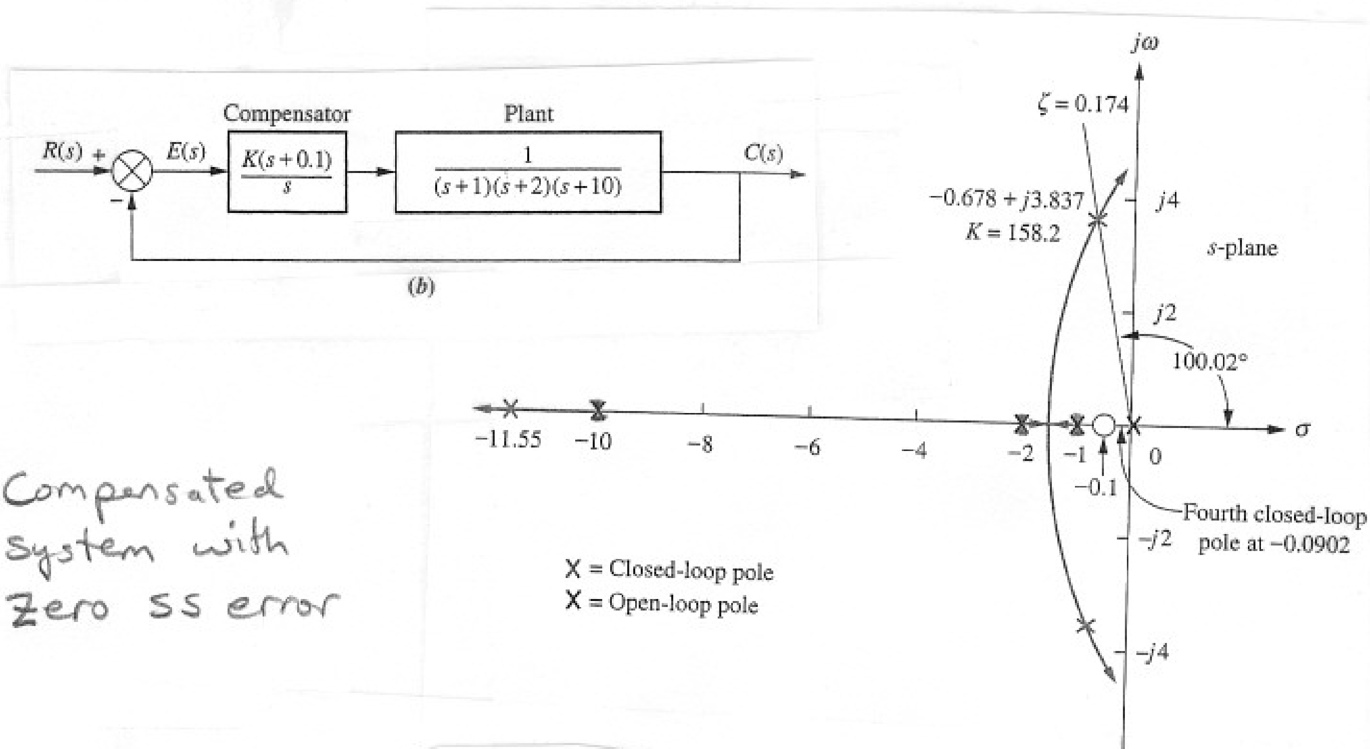
c. approximately on the root locus with compensator pole and zero added



Example

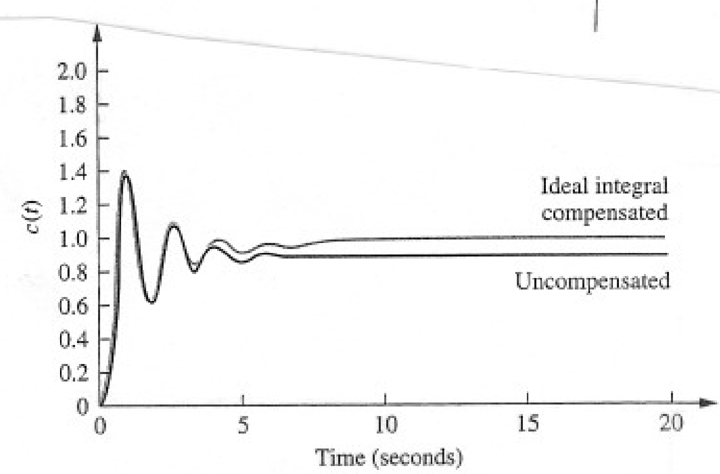


System with steady state error



Compensated system with zero SS error

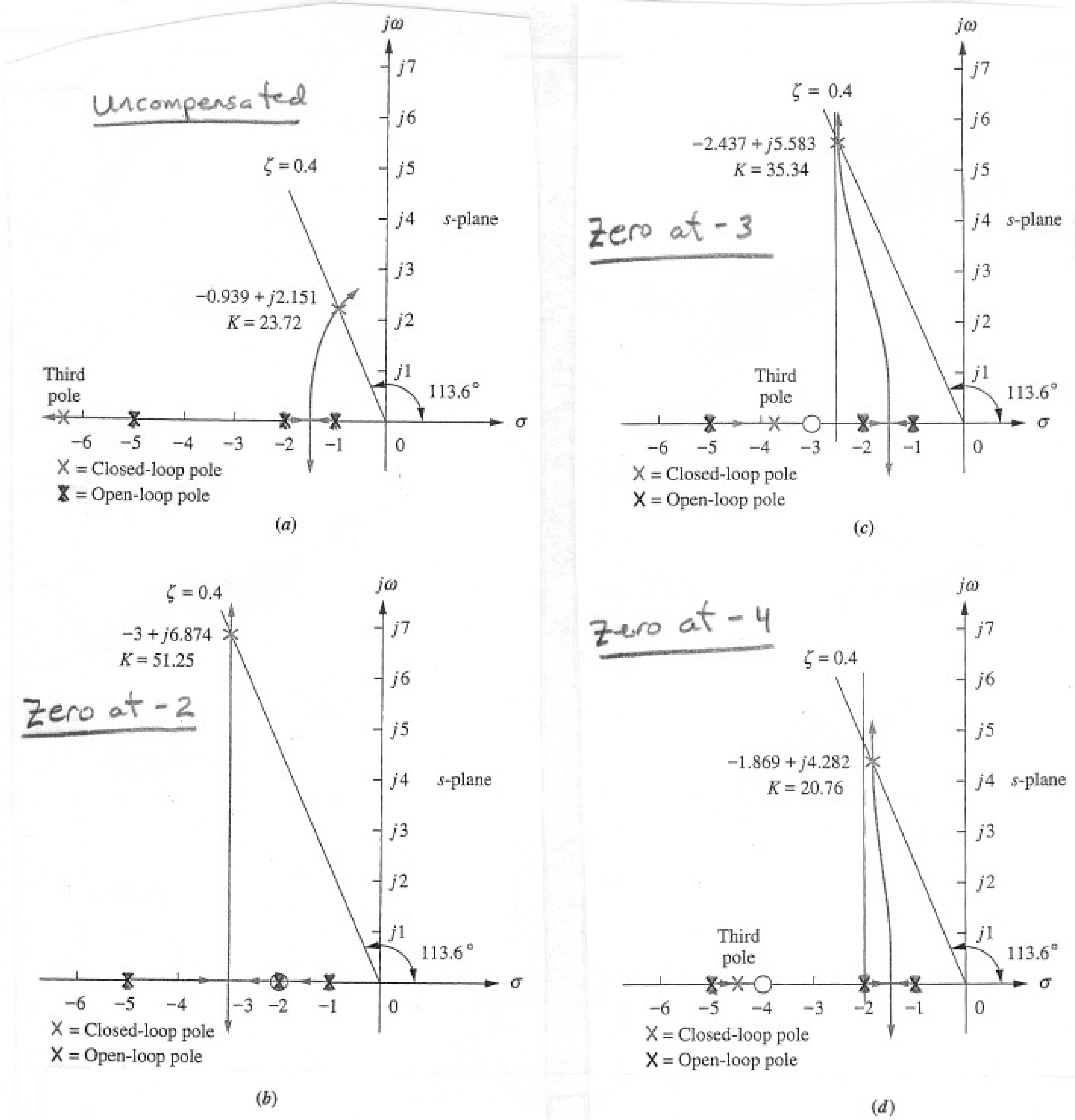
Ideal integral compensated system response and the uncompensated system response of Example 9.1



Proportional Derivative Compensation (PD)

One way to speed up the original system is to add a single zero to the forward path.

$$G_c(s) = s + z_c$$

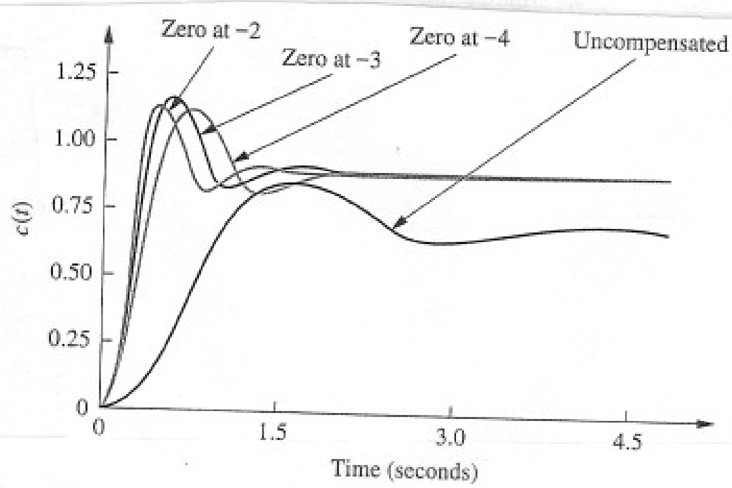


- The damping ratio is chosen to be 0.4 → %OS doesn't change
- The compensated poles have a more negative real value → t_s is shorter
- The compensated poles have a larger imaginary value → t_p is shorter

(Compared to the uncompensated case)

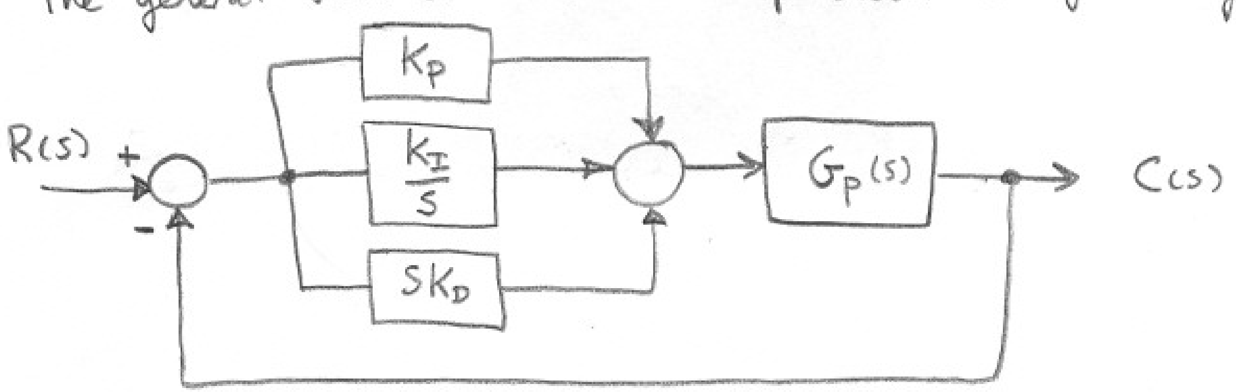
The corresponding values and step responses are:

	Uncompensated	Compensation b	Compensation c	Compensation d
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
$\%OS$	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	−6.123	None	−3.127	−4.262
Zero	None	None	−3	−4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK



Proportional Integral Derivative Compensation (PID)

The general form of a PID compensator is given by



Where the compensator is given by

$$G_c(s) = K_p + \frac{K_I}{s} + sK_D = \frac{K_p}{s}\left[s + T_i + T_d s^2\right]$$

where $T_i = \dfrac{K_I}{K_p}$ $\qquad T_d = \dfrac{K_D}{K_p}$

- The root locus plots the closed loop poles as a function of some varying parameter (or gain) for the system loop gain. For unity feedback the open loop gain is given by $G_c(s)\,G_p(s)$. Therefore we can plot the root locus for a PID controller by combining the plant and the compensator and letting K_p vary. (Assuming T_i and T_d are constant).
- If $G_p(s)$ is a second order plant then the open loop gain is given by:

$$\frac{K_p\left[s^2 T_d + s + T_i\right]\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

Now we can use the Matlab command "rlocus" to plot the root locus as K_p varies from 0 to ∞.

- From the root locus plot we can make conclusions about the system response and about stability.